Emmy Noether, Greatest Woman Mathematician

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Emmy Noether was born over one hundred years ago in the German university town of Erlangen, where her father, Max Noether, was a professor of mathematics. At that time it was very unusual for a woman to seek a university education. In fact, a leading historian of the day wrote that talk of “surrendering our universities to the invasion of women . . . is a shameful display of moral weakness.”[1] At the University of Erlangen, the Academic Senate in 1898 declared that the admission of women students would “overthrow all academic order.”[2]

In spite of all this, Emmy Noether was able to attend lectures at Erlangen in 1900 and to matriculate there officially in 1904. On 13 December 1907 she received her Ph.D. under the direction of Paul Gordan.

Transition: From Gordan to Hilbert

Gordan, near retirement at that time, was known as the “king of invariant theory,” a title he had earned because of his truly phenomenal ability to carry out symbolic calculations in his head. But Gordan had his limitations. As Max Noether put it, “Gordan was never able to do justice to the development of fundamental concepts; even in his lectures he completely avoided all basic definitions of conceptual nature, even that of the limit.”[3]

Thus it was essential to Emmy Noether’s development as a mathematician that Gordan’s successor, Ernst Fischer, influenced her away from Gordan’s type of research and toward the methods of David Hilbert, who is now widely regarded as the greatest mathematician since Gauss (see, for example, Reid 1970).

Noether's knowledge of invariant theory appeared useful to Hilbert in his work on Albert Einstein's theory of relativity, and she accepted an invitation to move to the University of Göttingen. In 1918 she presented her paper, *Habilitationsschrift*, on differential invariants, including a theorem now known as Noether's theorem. When Einstein read Noether's work, he wrote to Hilbert, "Yesterday I received from Miss Noether a very interesting paper on invariant forms . . . she certainly knows what she is doing."[4]

Noether's Theorem

Emmy Noether is famous for her work in two fields: physics and mathematics, or more specifically, Noether's theorem in physics, and the theory of ideals in algebra. Many algebraists who know about Noetherian rings have never heard of Noether's theorem, and most who have heard of it would be surprised to know how important it is.

It would be impractical to state the theorem here. However, its importance is suggested by physicist Peter G. Bergmann: "Noether's Theorem, so-called, forms one of the corner stones of work in general relativity as well as in certain aspects of elementary particles physics. The idea is, briefly, that to every invariance or symmetry property of the laws of nature (or of a proposed theory) there corresponds a conservation law. . . ."[5]

In Tavel (1971) the interested reader will find a translation of the 1918 publication in which Emmy Noether first proved Noether's theorem. The translation is introduced by an outline of a wide variety of applications. Noether's 1918 publication has also been translated in its entirety into Russian. Its importance is further manifested by the length of treatment of Noether's theorem in the *Great Soviet Encyclopedia.*

There is no absolute yardstick for measuring the relative importance of scientific works. However, one can glean from *Science Citation Index* a "first approximation," in some cases. It is certainly of interest that for the years 1965 to 1979 (August), the *Index* (which covers virtually all major mathematical and physical journals) lists 160 citations, by various authors, of Noether's 1918 publication. For the same years, the *Index* lists only 69 citations of all the rest of Noether's publications.

Noetherian Rings

From 1920 to 1926, Noether opened up new frontiers in algebra by her studies of ideals and by her effectiveness as a teacher of a "somewhat noisy and stormy" group of highly talented graduate students known as "the Noether boys."

The theory of ideals had been introduced a generation earlier by Richard Dedekind for the purpose of formulating the fundamental theorem of arithmetic (that every positive integer is a unique product of primes) for other numbers than just the integers. The genius of Dedekind's ideals is suggested by an example: Let (2) denote the set of even integers and (6) the set of all integer multiples of 6. Then the fact that 6 is divisible by 2 is equivalent to the containment $(6) \subseteq (2)$. More generally, n is divisible by m if and only if $(m) \subseteq (n)$. It follows that *everything about divisibility can be formulated in terms of set containment!* The sets (n) are examples of ideals in the *ring* of integers.

The theory of ideals had been developed considerably before 1920. But many proofs, such as that of Lasker's theorem (named for Emanuel Lasker, algebraist and world chess champion) could be replaced by much simpler proofs only after Noether showed the importance of a particular condition for rings. It is called the *ascending chain condition* (ACC). To define the ACC, we first define a set of ideals $I_1, I_2, I_3, \ldots$ in a ring R to be an ascending chain if $I_1 \subseteq I_2 \subseteq I_3 \subseteq \ldots$. Now, if *every* ascending chain in R comes to an end (meaning that there is some k such that $I_k = I_{k+1} = I_{k+2} = \ldots$), then R has the ACC. In 1921, Noether published a paper showing the naturalness and usefulness of the ACC, and it is largely as a result of that paper that rings satisfying the ACC are called *Noetherian rings*. The integers, with the ordinary $+$ and $\cdot$, for example, are a Noetherian ring.

Moderne Algebra

B. L. van der Waerden came to Göttingen in 1924 after finishing the university course in his native Holland. He readily grasped Noether's theories and, more than anyone else, enhanced them with his own discoveries and promulgated her work.

In 1930, van der Waerden published his famous book, *Moderne Algebra*, based on the lectures of Noether and Emil Artin. Many mathematicians have written about the significance of this book; for example, Garrett Birkhoff wrote that "van der Waerden made 'modern algebra' suddenly seem central in mathematics."[6]

"It is not too much to say," continued Birkhoff, "that the freshness and enthusiasm of his exposition electrified the mathematical world—especially mathematicians under 30 like myself." Translated into English and reprinted many times, the book has brought the influence of Noether into the American classroom.

The conceptual axiomatic style of mathematical thinking for which Noether is so often remembered led Carroll V. Newsom to write, regarding modern algebra, in his introduction to the Twenty-third NCTM Yearbook (1957): ". . . major credit for its development should probably be given to the great woman mathematician, Emmy Noether. The work, *Moderne Algebra*, by . . . B. L. van der Waerden, provides the classical account of modern algebra as Emmy Noether conceived it."

Mecca of Mathematics

While Noether was in Göttingen, the Mathematische Institut there became known informally as the "Mecca of Mathematics." Perhaps there has never been a mathematics department composed of a more distinguished faculty, attracting such a stream of visiting scholars from around the world as the one in Göttingen during the years 1923 to 1933. Within this setting, by the early thirties, the circle of algebraists around Noether had gained recognition as the most active part of the Institut. What a loss it was in 1933, with the takeover by the Nazi party under Adolf Hitler, that the excellence at Göttingen was suddenly scattered into the four winds.

With the help of the Rockefeller Foundation, Noether was able to find refuge in America. She taught at Bryn Mawr College during the academic year 1933–34 and lectured weekly at the Institute for Advanced Study in nearby Princeton. It is a little-known fact that she was a member of the American Mathematical Society.

In support of Noether's continued support from the Rockefeller Foundation, Norbert Wiener, so often remembered today as the "father of cybernetics," wrote in 1935: "Miss Noether is a great personality; the greatest woman mathematician who has ever lived; and the greatest woman scientist of any sort now living, and a scholar at least on the plane of Madame Curie."[7] Noether's stature as the greatest woman mathematician is also indicated in writings by Hermann Weyl and Albert Einstein.

The Hamburg Untergrund

I have talked with many who knew Emmy Noether personally. There are two things they were all quick to mention. One is that Noether's students seemed always to be flocked around her, "following her as if following the Pied Piper," and the other is that she very seldom wrote or said anything that was not mathematical. The latter is amusingly illustrated by a short trip on the Hamburg *Untergrund* in 1934.

During that summer, Noether had returned to Germany and was visiting the Artins in Hamburg. It was Emil Artin whose lectures, along with Noether's, had served as a basis for van der Waerden's *Moderne Algebra*. In 1980, it was my pleasure to interview Mrs. Artin, now Natascha Artin Brunswick, and I asked what she remembered about the visit. Here is what she said:

> "Now the one thing I remember most vividly is the trip on the Hamburg *Untergrund*, which is the subway in Hamburg. We picked up Emmy at the Institute, and she and Artin immediately started talking mathematics. At that time it was Idealtheorie, and they started talking about *Ideal, Führer,* and *Gruppe*, and *Untergruppe*, and the *whole car* suddenly started pricking up their ears. [Each of the German nouns has both mathematical and political meanings.] And *I* was frightened to death—I thought, my goodness, next thing's going to happen, somebody's going to arrest us. Of course, that was in '34, and all. But Emmy was *completely* oblivious, and she talked very loud and very excited, and got louder and louder, and all the time the "*Führer*" came out, and the "*Ideal*." She was *very* full of life, and she constantly talked very fast and very loud."

"Warm Like a Loaf of Bread"

There are many aspects of Noether's life and work that can only be suggested here: her students, many of whom became leading mathematicians; her manner of teaching—what she lacked as a lecturer was more than equalled by her buoyant spirit and generous sharing of powerful original insights; her research; the inadequacy of recognition and remuneration during her lifetime because she was a woman; her family; and her remarkable personality—she was "warm like a loaf of bread," wrote Hermann Weyl, ". . . there irradiated from her a broad comforting, vital warmth."[8]

To learn more about the greatest woman mathematician, the interested reader will certainly want to take a look at two of the books listed in the bibliography, Brewer and Smith (1962) and Dick (1981).

Notes

1. Heinrich von Treitschke, as translated in Richard J. Evans, *The Feminist Movement in Germany 1894–1933* (London: Sage, 1976), p. 17.
2. *The Feminist Movement in Germany,* p. 18.
3. Max Noether, "Paul Gordan," *Mathematics Annual* 75 (1914): 1–41.
4. Einstein to Hilbert, 24 May 1918, in Einstein archive at the Institute for Advanced Study.
5. Reprinted from Clark Kimberling, "Emmy Noether," *American Mathematical Monthly* 79 (1972): 136–49.
6. Garrett Birkhoff, "Current Trends in Algebra," *American Mathematical Monthly* 80 (1973): 760–82; correction, 81 (1974): 746.
7. Wiener to Jacob Billikopf, 2 January 1935, in the Rockefeller Archive Center.
8. Hermann Weyl, "Emmy Noether," *Scripta Mathematica* 3 (1935): 201–20; reprinted in *Gesammelten Abhandlungen,* III, Springer, 1968 and in Dick, 1981.

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